Regression Models for Ordinal Data
Introducing *R*-package **ordinal**

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Examples of ordinal response variables

- MR scannings of cancer (greatly enlarged, enlarged, no change, smaller, much smaller)
- Smoking frequency (never, occasionally, <1 pack/day, >1 pack/day)
- BMI (underweight, normal weight, overweight, obese)
- Questionaire (strongly disagree, disagree, undecided, agree, strongly agree)

Cumulative link models (CLMs)

The cumulative link model — also known as:

- Proportional odds model
- Ordered probit/logit model
- Ordinal regression model

$$\text{CLM:} \quad P(Y_i \leq j) = g(\theta_j - \boldsymbol{x}_i^T \boldsymbol{\beta})$$

The wine data

Table: The wine data (Randall, 1989), N=72

Variables	Type	Values
bitterness	response	1, 2, 3, 4, 5
		less — more
temperature	predictor	cold, warm
contact	predictor	no, yes
judges	random	1, . . . , 9

- How does the perceived bitterness of wine depend on temperature and contact?
- A linear model is not a good idea

The **ordinal** package — an overview

Main functions:

- Cumulative link models (CLMs):

```
clm(formula, data, link, ......)
```

- Cumulative link mixed models (CLMMs):

```
clmm(formula, data, link, ......)
```

  (lmer syntax)

Other functions:

- `clm.control`
- `clmm.control`
- 15 additional exported function

Numerous methods:

- `summary`, `anova`, `predict`, `confint`, ...

Existing implementations of cumulative link models

- `polr` from **MASS** — widely used implementation
- `lrm` from **Design**
- `cumulative` from **VGAM**
- `MCMCglmm` from **MCMCglmm** (mixed models)

Challenges in implementing CLMs

1. Intuitive user interface
2. Efficient computational methods
3. Substantial scope of models
4. Useful methods and auxiliary functions
5. Readable code
6. Comprehensive Documentation

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Fitting and displaying CLMs with ordinal

```
> fm1 <- clm(rating ~ temp + contact, data = wine, link = "probit")
> summary(fm1)
formula: rating ~ temp + contact
data:    wine

 link   threshold nobs logLik AIC    niter max.grad cond.H
 probit flexible  72   -85.76 183.52 5(0)  1.44e-13 2.2e+01

Coefficients:
           Estimate Std. Error z value Pr(>|z|)
tempwarm     1.4994     0.2918   5.139 2.77e-07 ***
contactyes   0.8677     0.2669   3.251  0.00115 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Threshold coefficients:
    Estimate Std. Error z value
1|2  -0.7733     0.2829  -2.734
2|3   0.7360     0.2499   2.945
3|4   2.0447     0.3218   6.353
```

Aliased coefficients

```
> fm.soup <- clm(SURENESS ~ PRODID * DAY, data = soup)
> summary(fm.soup)
formula: SURENESS ~ PRODID * DAY
data:    soup

 link  threshold nobs logLik   AIC     niter max.grad cond.H
 logit flexible  1847 -2672.08 5374.16 6(1)  1.95e-13 9.4e+02

Coefficients: (1 not defined because of singularities)
             Estimate Std. Error z value Pr(>|z|)
PRODID2        0.6665     0.2146   3.106  0.00189 **
PRODID3        1.2418     0.1784   6.959 3.42e-12 ***
PRODID4        0.6678     0.2197   3.040  0.00237 **
PRODID5        1.1194     0.2400   4.663 3.11e-06 ***
PRODID6        1.3503     0.2337   5.779 7.53e-09 ***
DAY2          -0.4134     0.1298  -3.186  0.00144 **
PRODID2:DAY2   0.4390     0.2590   1.695  0.09006 .
PRODID3:DAY2       NA         NA      NA       NA
PRODID4:DAY2   0.3308     0.3056   1.083  0.27892
PRODID5:DAY2   0.3871     0.3248   1.192  0.23329
```

Likelihood ratio tests of CLMs

```
> fm2 <- update(fm1, ~. - temp)
> anova(fm1, fm2)
Likelihood ratio tests of cumulative link models:

    formula:                link:  threshold:
fm2 rating ~ contact        probit flexible
fm1 rating ~ temp + contact probit flexible

    no.par    AIC   logLik LR.stat df Pr(>Chisq)
fm2      5 210.05 -100.026
fm1      6 183.52  -85.761  28.529  1  9.231e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Computational challenges

- Robust starting values
  - The `clm` should always converge from the default starting value
  - It should be possible to supply starting values
- Speedy model estimation
  - Speed is maintained despite model scope and flexibility
- Accurate estimates
- Accurate standard errors

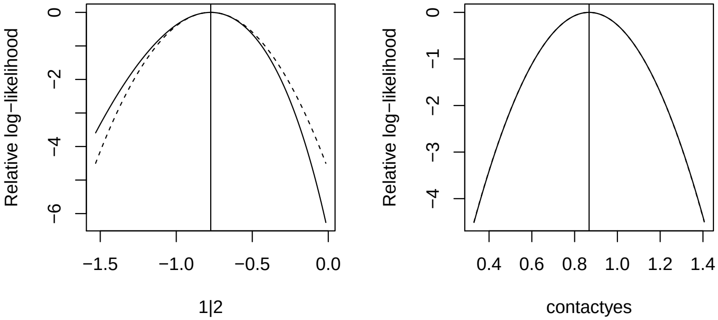
Accuracy of parameter estimates

```
> fm1
formula: rating ~ temp + contact
data:    wine

 link   threshold nobs logLik AIC    niter max.grad
 probit flexible  72   -85.76 183.52 5(0)  1.44e-13

Coefficients:
  tempwarm contactyes
    1.4994     0.8677

Threshold coefficients:
    1|2     2|3     3|4     4|5
-0.7733  0.7360  2.0447  2.9413
```

- Has the model converged?
- How accurate are these estimates?

Assessment of model convergence

```
> slice.fm1 <- slice(fm1, parm = c(1, 6))
> par(mfrow = c(1, 2))
> plot(slice.fm1)
```



Assessment of parameter accuracy

```
> convergence(fm1)
 nobs logLik niter max.grad cond.H  logLik.Error
 72   -85.76 5(0)  1.44e-13 2.2e+01 0.00e+00

           Estimate Std.Err  Gradient      Error Cor.Dec Sig.Dig
1|2         -0.7733  0.2829  1.59e-14  1.91e-16      15      15
2|3          0.7360  0.2499  1.31e-13 -5.65e-16      14      14
3|4          2.0447  0.3218 -1.44e-13 -8.26e-15      13      14
4|5          2.9413  0.3873 -6.46e-15 -7.72e-15      13      14
tempwarm     1.4994  0.2918 -1.38e-14 -5.00e-15      14      15
contactyes   0.8677  0.2669  1.88e-15 -2.25e-15      14      14

Eigen values of Hessian:
61.616 53.876 32.283 17.241 13.393  2.825
```

Extending the model class

- Scale effects

```
clm(rating ~ contact, scale =~ temp, data=wine)
```

- Structured thresholds

```
clm(rating ~ contact, data=wine, threshold="symmetric")
clm(rating ~ contact, data=wine, threshold="equidistant")
```

- Nominal effects (partial proportional odds)

```
clm(rating ~ contact, nominal =~ temp, data=wine)
```

- Flexible link functions
- Random effects
  - For grouped and multilevel data

Cumulative link mixed models (CLMMs)

- Multiple random effect terms
  - Nested and crossed random effect structures
  - No correlated random effects (yet)
  - No random slopes (yet)
- Computational methods
  - Laplace approximation
  - Adaptive Gauss-Hermite quadrature (+ non-adaptive GHQ)

Example:

```
> fm.ran <- clmm(rating ~ contact + temp + (1 | judge), data = wine)
```

Methods for `clm` fits

- Standard methods:
  `print`, `summary`, `anova`, `predict`
- Extractor methods:
  `coef`, `vcov`, `logLik`, `AIC`, `fitted`, ...
- Model development and selection:
  `drop1`, `add1`, `step`
- Model assessment methods:
  `profile`, `plot.profile`, `confint`
- Numerous additional methods

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Summary

- Reliable computational methods
- Methods for assesing convergence
- Extends the basic model with:
  - scale effects
  - nominal effects
  - random effects
  - structured thresholds
- A suite of helpful methods for `clm` and `clmm` objects

Future work

- `slice` and `convergence` methods for `clmm` fits
- More flexible random effect structures
- AGQ methods for nested random effects

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Bibliography

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