Weierstrass Institute for
Applied Analysis and Stochastics

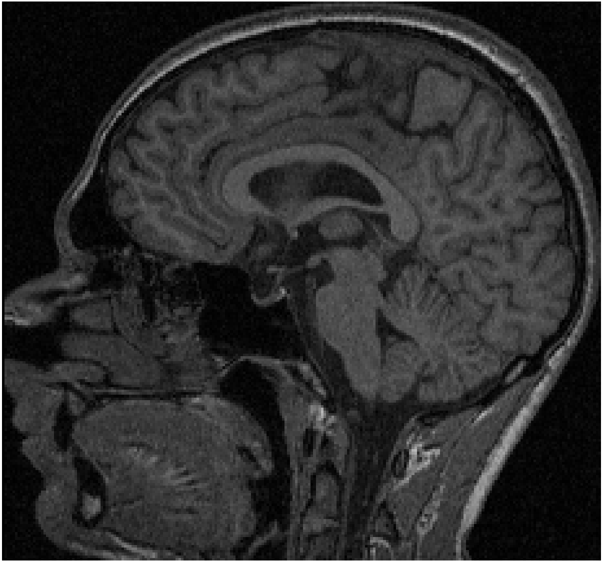
DFG-Forschungszentrum MATHEON
Mathematik für Schlüsseltechnologien

Statistical Parametric Maps for Functional MRI Experiments in R: The Package fmri

Karsten Tabelow

UseR!2011

Images and Noise



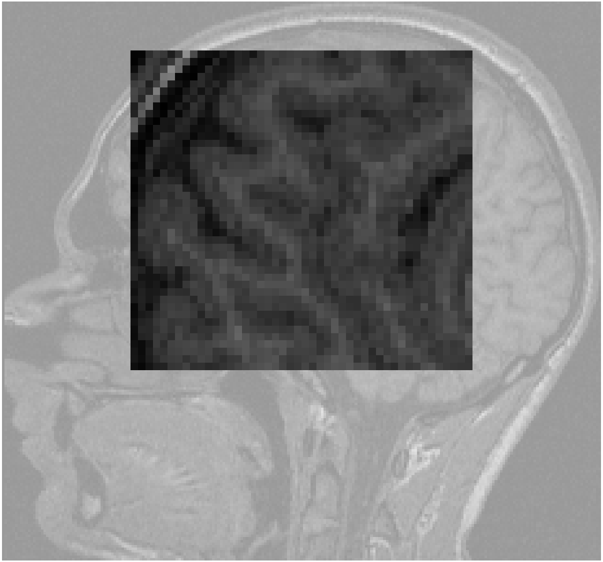


Image Denoising Methods

- Kernel estimators
- Wavelets, Curvelets, ...
- MCMC, SANN
- Diffusion methods
- Scale-space methods
- Scanner upgrade

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Structural Adaptive Smoothing

- Local parametric model for data
- Iterative estimation of parameters and homogeneity regions
- Dimension-free, structure preserving/enhancing

General setup / Local parametric model

- Design: $x_1, \ldots, x_n \in \mathcal{X} \subseteq I\!R^p$
- Observations: $Y_1, \ldots, Y_n \in \mathcal{Y} \subset I\!R^q$

$$Y_i \sim \mathbf{P}_{\theta(x_i)} \qquad \theta : I\!R^p \rightarrow \Theta \quad \text{(i.i.d.)}$$

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Structural assumption

- $\exists$ Partitioning $\mathcal{X} = \bigcup_{m=1}^{M} \mathcal{X}_m$ such that

$$\theta(x) = \theta(x_i) \Leftrightarrow \exists m : x \in \mathcal{X}_m \wedge x_i \in \mathcal{X}_m$$

  i.e. θ constant on each $\mathcal{X}_m$
- Some components of θ may be global parameters.

General idea

- Determine *structure* and *estimates* in an *iterative* procedure
- Weighting scheme $W(x_i) = (w_{i1}, \dots, w_{in})$
- $W(x_i)$ describes a local model

$$w_{ij}^{(k)} = K_{loc}(||x_i - x_j||^2 / h_k^2) \quad \cdot \quad K_s(s_{ij}^{(k)} / \lambda)$$

s_{ij} measures the difference between estimates $\hat{\theta}(x_i)$ and $\hat{\theta}(x_j)$

- Four steps:
  - A) Initialize estimates $\hat{\theta}(x_i)$ and describe initial partitioning $W(x_i)$ ideally by observed values and $w_j(x_i) = w_{ij} = \delta_{ij}$
  - B) “Learn” about $\theta(x)$ from information on partitioning
  - C) “Learn” about partitioning from estimates $\hat{\theta}(x)$
  - D) Iterate while inspecting scale space from local to global

Results

- *Propagation* under homogeneity: For a homogeneous situation and $\mu > 4$ the last step estimate $\hat{\theta}_i^{k^*}$ fulfills $\boldsymbol{P}\big(\bar{N}_i^{(k^*)}\,\mathcal{K}(\hat{\theta}_i, \theta) > \mu \log(n)\big) \leq 2k^*/n.$
- *Propagation* under local homogeneity: Similar results for interior points of local homogeneous regions.
- *Stability* of estimates: The quality of the best intermediate result holds (up to a constant) for the final estimate.
- *Optimal adaptive rate* of estimation under smoothness conditions on f_θ.
- *Separation* property

Parameters

- λ can be selected by a *propagation condition*, independ of data
- k^* determines the maximum bandwidth
- $c_h = 1.25^{1/p}$ provides exponential growth of sum of location weights
- Kernels $K_{loc}(z) = (1-z)_+$ and $K_s(z) = \min(1, 2(1-z))_+$

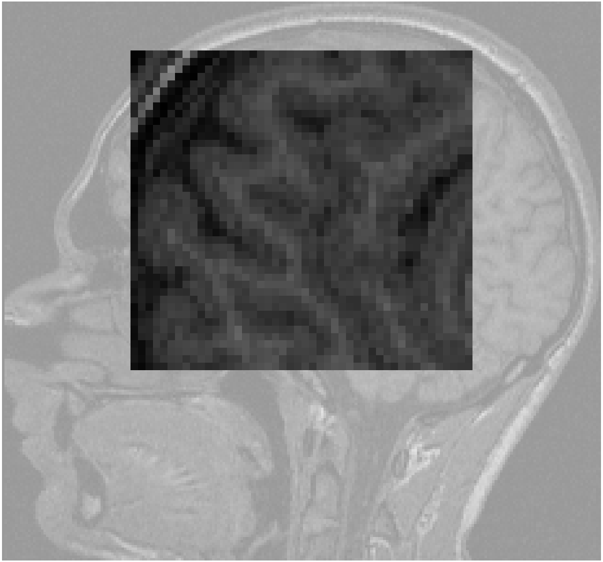
Propagation condition

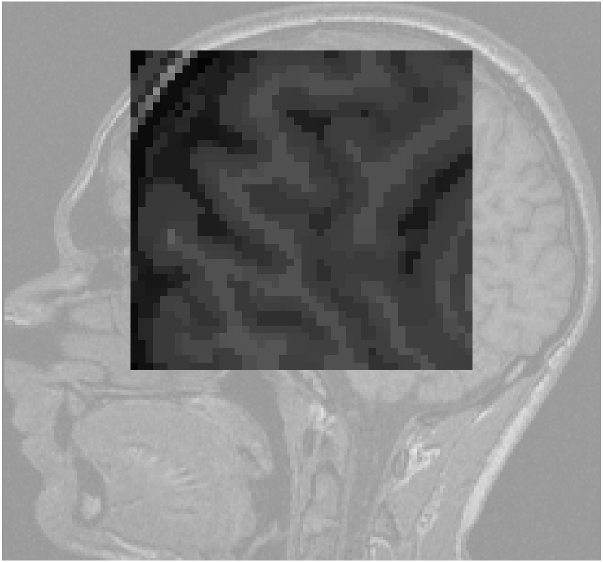
$\hat{\theta}^{(k)}(x)$	PS estimate of $\theta(x)$	from iteration k,
$\tilde{\theta}^{(k)}(x)$	Kernel estimate of $\theta(x)$	using bandwidth $h^{(k)}$

Let $\theta(x) \equiv \theta$. Select parameters such that $\forall k$ and some $\alpha > 0$

$$\mathbf{E}\,|\hat{\theta}^{(\mathbf{k})}(\mathbf{x}) - \theta| \leq (\mathbf{1} + \alpha)\,\mathbf{E}\,|\tilde{\theta}^{(\mathbf{k})}(\mathbf{x}) - \theta|.$$

Images and Noise





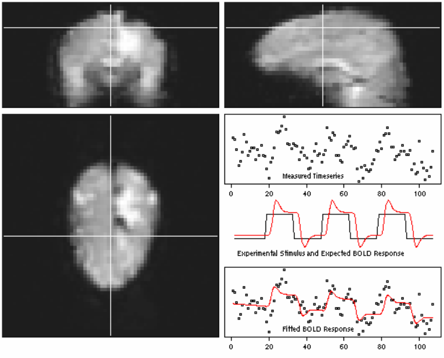
Human Eye vs. Structural adaptive smoothing

- Human eye is a very good denoiser
- Experience: Structural adaptive smoothing compares well with eyes.
- Limited use for visual inspection and for volumetric (integral) quantities

Applications

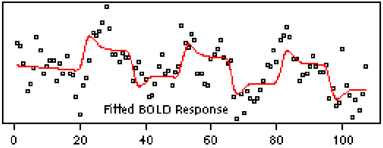
- Suitable for higher dimensional data
- Structure enhancement, structure identification.
- functional MRI - package **fmri**

fMRI data = 3D + T



How to smooth fMRI data?

- Blurring effect in Gaussian filter
- T2-volumes have limited structure.
- Local tests inefficient in 3D+T space
- Interesting structure is activation structure
- We have a model about HRF



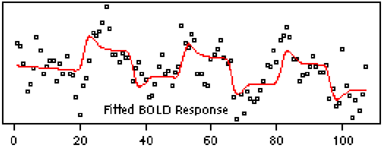
Data Preparation

Registration

Motion correction

Normalization

Smoothing



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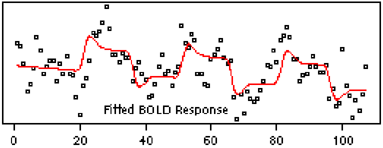
Linear Modeling

$Y = X\beta + \epsilon$

Prewhitening

$\tilde{Y} = \tilde{X}\beta + \tilde{\epsilon}$



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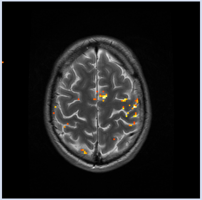
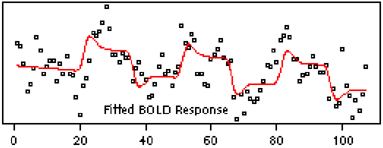


no filter

Thresholding

t-statistic ...

multiple test problem



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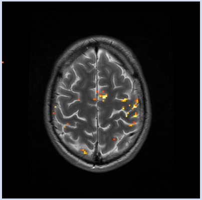

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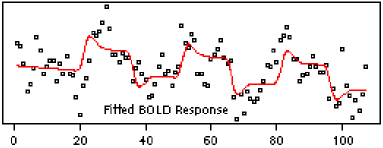
FDR

fMRI data analysis



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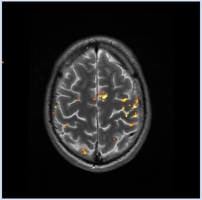

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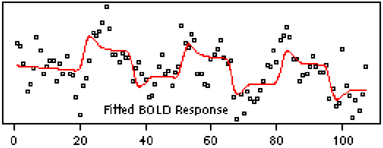
Thresholding

t-statistic

RFT



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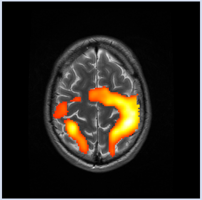


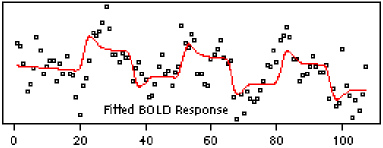
Gaussian filter

Thresholding

t-statistic

RFT

fMRI data analysis



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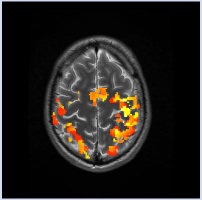
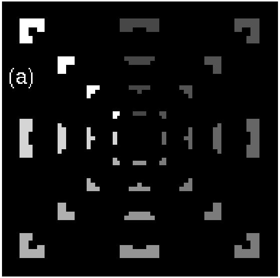


adaptive!

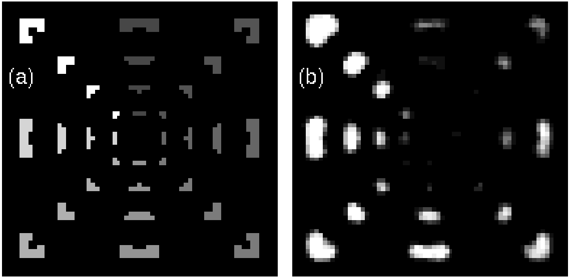
Thresholding

t-statistic

RFT



(a) Signal location in Phantom data (3D) with different signal size
26 slices with 15 containing activations



(a) Signal location in Phantom data (3D) with different signal size
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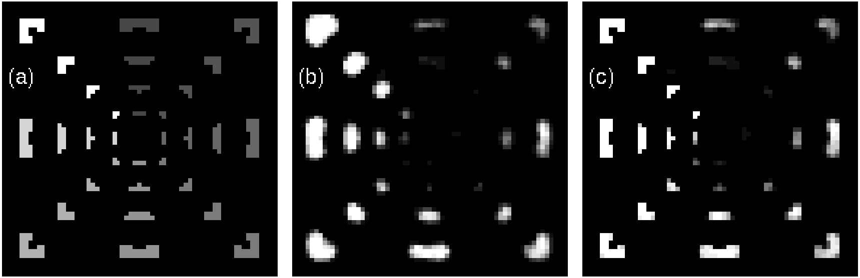
(b) Relative proportion of detection using non adaptive smoothing

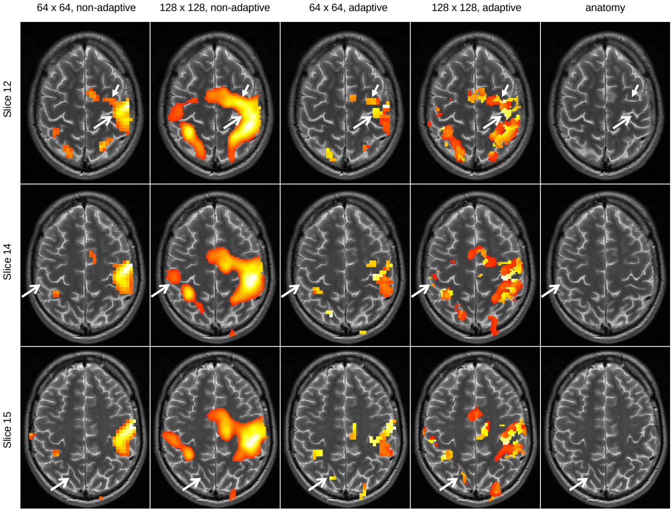


(a) Signal location in Phantom data (3D) with different signal size
26 slices with 15 containing activations

(b) Relative proportion of detection using non adaptive smoothing

(c) Relative proportion of detection using Structural Adaptive Smoothing

Finger tapping fMRI data at different resolutions



How to integrate smoothing and testing in on step?

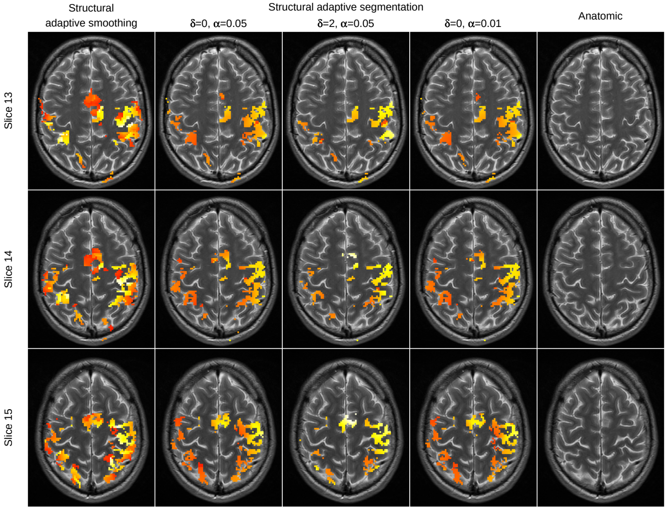
$$H : \max_{i \in V} \gamma_i \leq \delta \qquad (\text{or} \quad \max_{i \in V} |\gamma_i| \leq \delta).$$

$$T(\hat{\Gamma}^{\mathcal{H}}) = \max_{h \in \mathcal{H}} \max_{i \in V} \frac{\left(\hat{\gamma}_i^{(h)} - \delta\right)}{a_{n(h)}(\nu) \sqrt{\hat{\boldsymbol{D}} \hat{\gamma}_i^{(h)}}} - \frac{b_{n(h)}(\nu)}{a_{n(h)}(\nu)}$$

Structural adaptation

- Distribution of $T(\hat{\Gamma}^{\mathcal{H}})$ under hypotheses by simulation, determine critical values
- Inspect scale space sequentially and segment as soon as critical value is exceeded
- Use adaptive smoothing as long as Hypothesis not rejected

Structural adaptive segmentation: Results



FMRI analysis

```
library(fmri);library(adimpro);

ds <- read.NIFTI("Imagination.nii")
anatomic <- extract.data(ds)[,,,1]

scans <- 105
onsets <- c(16, 46, 76)
duration <- 15
tr <- 2
hrf <- fmri.stimulus(scans, onsets, duration, tr)
x <- fmri.design(hrf)
spm <- fmri.lm(ds, x)

spm.segment <- fmri.smooth(spm, hmax = 4, adaptation = "segment")

ds.ana <- read.NIFTI("MPRAGEco.nii")
for (slice in 2:30) {
  img <- plot(spm.segment, ds.ana, slice = slice)
  write.image(make.image(img, gammatype="ITU"), file=paste("result", slice, ".png", sep=""))
}
```

Package fmri

- Structural adaptive smoothing for fMRI (using RFT)
- Structural adaptive segmentation for fMRI
- Package **fmri**: I/O (cf. **oro.nifti**)
- Package **fmri**: Linear Modeling
- Package **fmri**: Smoothing
- Package **fmri**: Signal detection
- Package **fmri**: Publication ready images
- Package **fmri**: No group analysis
- Cf. packages **arf3DS4**, **AnalyzeFMRI**, **neuRosim**, **cudaBayesreg**

Collaborations

Joint Work with:

- Jörg Polzehl and Vladimir Spokoiny, WIAS and MATHEON
- Henning U. Voss, Weill Medical College, Cornell University

Cooperation:

- Citigroup Biomedical Imaging Center, Weill Medical College, Cornell University
- BNIC, Charité, Berlin

R-Community:

- CRAN Task View: Medical Image Analysis

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